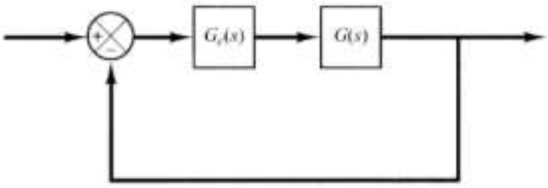
Control System Design: Lead Compensator

Control system diagram in unity feedback



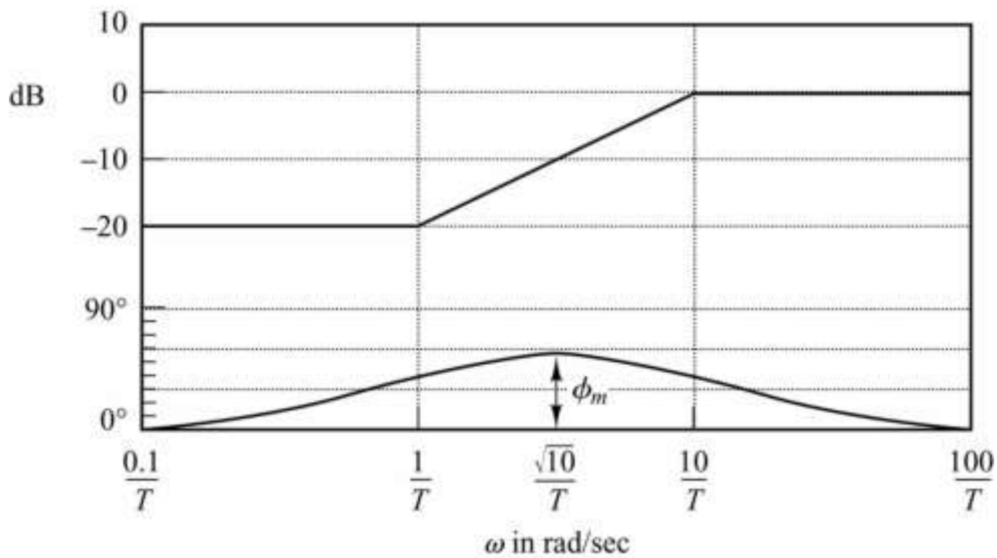
$G_C(s)$ – Compensator / Controller; $G(s)$ – Plant / Transfer function

Lead compensation techniques based on the frequency response approach

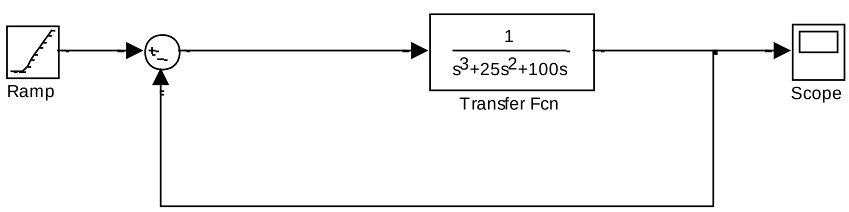
Lead compensator transfer function

$$G_c(s) = K_c \alpha \frac{Ts+1}{\alpha Ts+1} = K_c \frac{s+1/T}{s+1/\alpha T}; \qquad 0 < \alpha < 1$$

Compensate magnitude and phase profile shown in figure



Example Lead design ; Desired system is K_v of 20 sec^{-1}, 15% overshoot related to PM of 53 degree (damping ratio of 0.516)



Determine and analysis of previous information

Open-loop TF is

; Type_______

Closed-loop TF is

Closed-loop poles are ____

Bandwidth frequency $(\omega_{BW}) =$ ____________rad/sec

Gain margin(GM) =____________ dB; Phase margin(PM) = ______________ degree

Static velocity error constant (K_v) = ________________ sec^{-1}

Settling time = ___________________________sec (5% error)

<u>Step I:</u> Determine total gain (K) of open-loop TF to satisfy the requirement on the given static velocity error constant (K_v)= 20

$$K_v = \lim_{s\to 0} sG_c(s)G(s) = \lim_{s\to 0} s\left(K_c\alpha\frac{Ts+1}{\alpha Ts+1}\right)\left(\frac{1}{(s+20)(s+5)s}\right) = 20$$

where $K_c\alpha = K$, thus

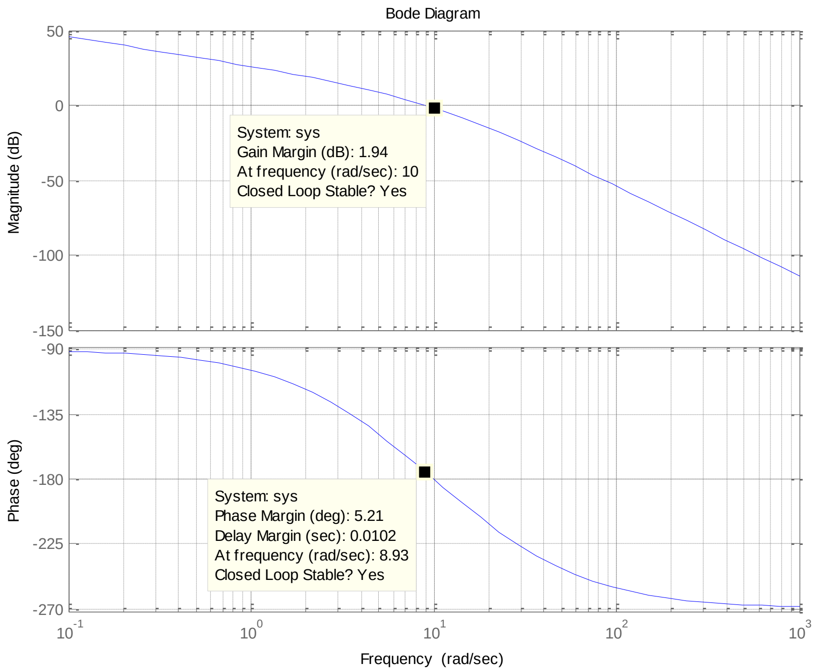
$$K = 20(100) = 2000$$

New open-loop transfer function

$$G_0(s) = \frac{2000}{(s+20)(s+5)s} = \frac{2000}{s^3+25s^2+100s}$$

Step II: Plot bode diagram of open-loop TF with new gain such as

$$G_0(s) = \frac{2000}{(s+20)(s+5)s} = \frac{2000}{s^3+25s^2+100s}$$



Phase margin(PM)= 5.2 deg. at 8.93 rad/sec ; Gain margin(GM)= 1.94 dB at 10 rad/sec

Step III: Phase margin requirement is 53 deg. plus 10 deg. Total PM is 63 deg.

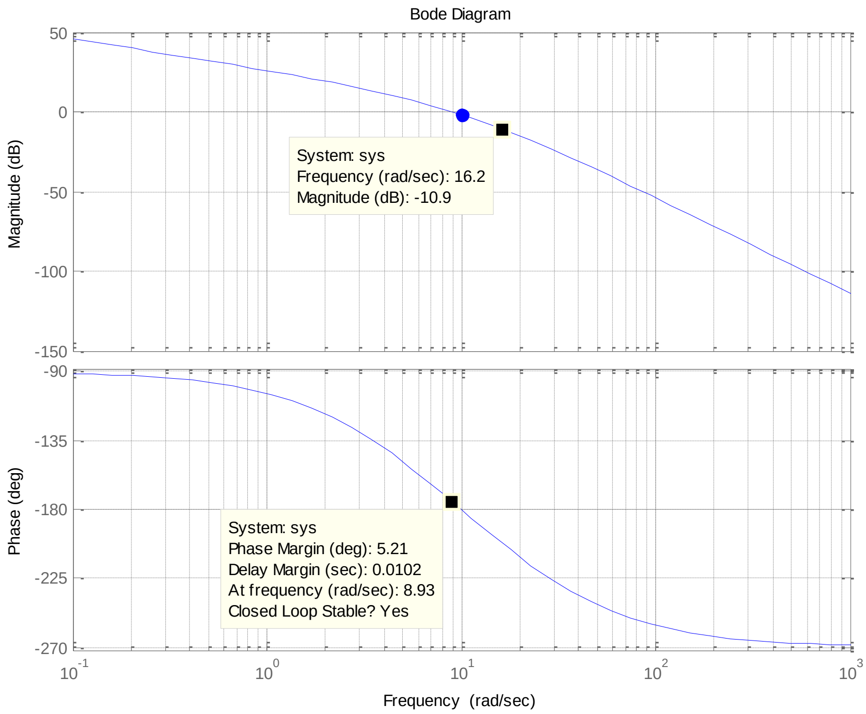
Now we have PM of 5 deg. and needs to add 58 deg.

Step IV: Determine , $\sin \varphi_{max} = \frac{1-\alpha}{1+\alpha}$

$$\varphi_{max} = 58 = sin^{-1}\frac{1-\alpha}{1+\alpha};$$
$$\alpha = 0.082$$

Step V: From $-20\log\frac{1}{\sqrt{\alpha}} = -20\log\frac{1}{\sqrt{0.082}} = -10.86\ \text{dB}$ at ω_{max}

Select ω_{max} to be new phase crossover frequency at this frequency must be – 10.86 dB. At 16.2 rad/sec is ω_{max}



$$\omega_{max} = \frac{1}{T\sqrt{\alpha}} = 16.2 \quad rad/sec$$

Thus T= 0.21; $\alpha = 0.082$

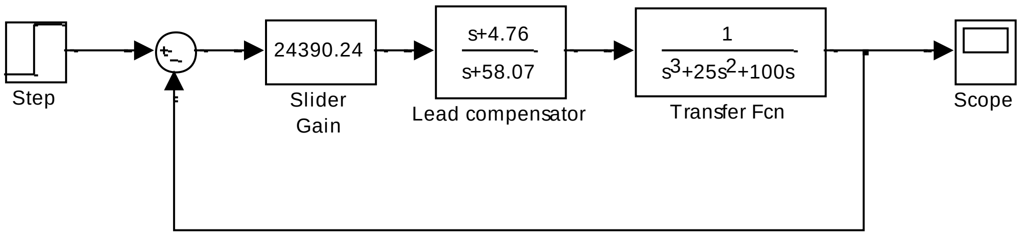
Now lead compensator is

$$G_c(s) = K_c \frac{s + 1/T}{s + 1/\alpha T} = K_c \left(\frac{s + 4.76}{s + 58.07}\right)$$

Step VI: Determine gain of lead compensator

$$K_c \alpha = K = 2000 \rightarrow K_c = 24390.24$$

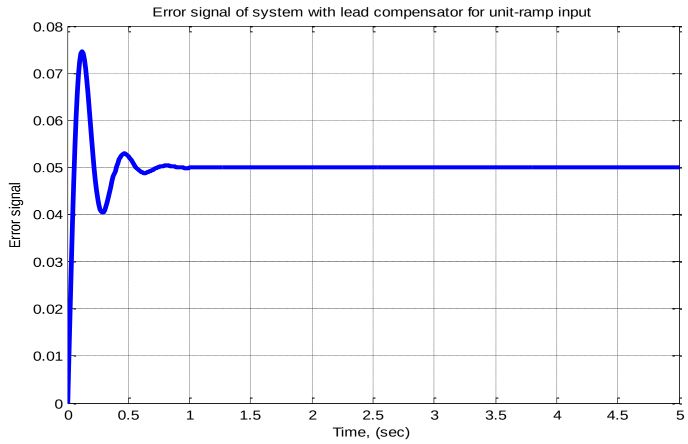
Now lead compensator is $G_c(s) = 24390.24 \left(\frac{s+4.76}{s+58.07}\right)$

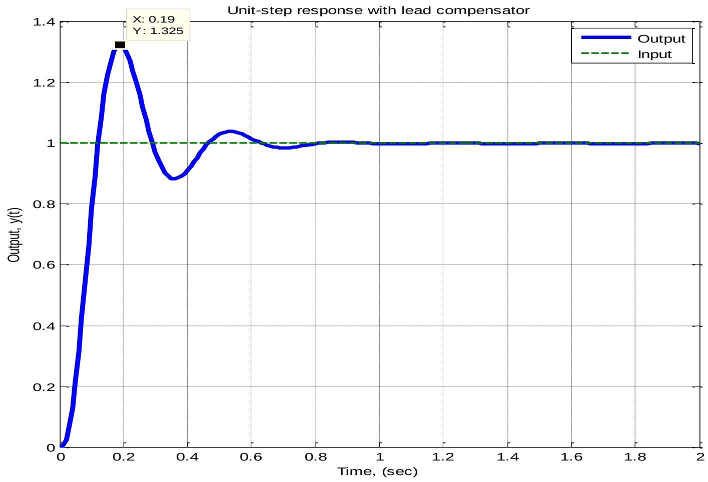


Check steady state error for unit-ramp input relation with static velocity error constant and PM relation with damping ratio (% overshoot)

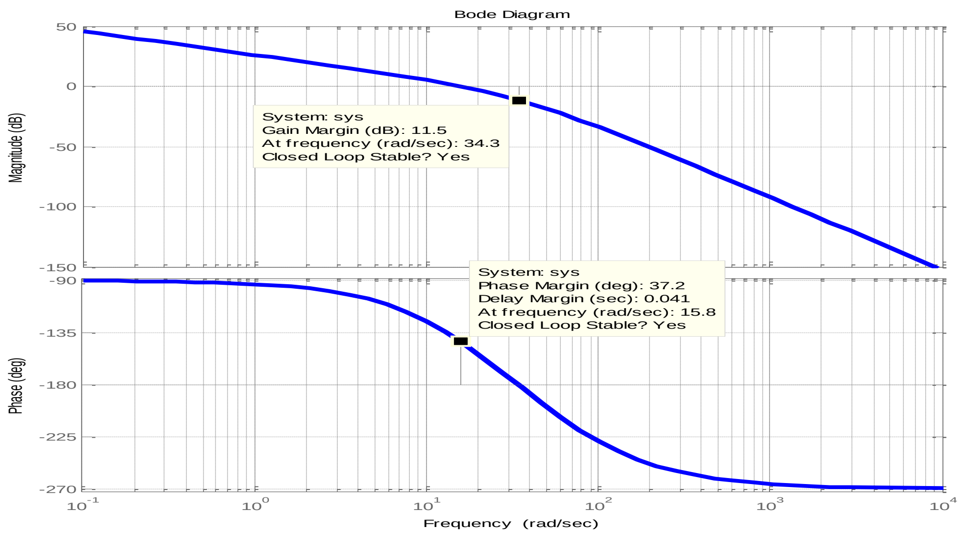
- Steady state error for unit-ramp input is 0.05 (Static velocity error constant is 20 sec^{-1})

- **% overshoot is 32.5** ....Desired system is 15% overshoot





Bode diagram of Open-loop TF $G_c(s)G(s) = 24390.24\left(\frac{s+4.76}{s+58.07}\right)\left(\frac{1}{(s+20)(s+5s)}\right)$



Phase margin(PM)= 37.22 deg. at 15.8 rad/sec ; Gain margin(GM)= 11.5 dB at 34.3 rad/sec

Redesign:

Step III: Phase margin requirement is 53 deg. plus 12 deg. Total PM is 65 deg.

Now we have PM of 5 deg. and needs to add 60 deg.

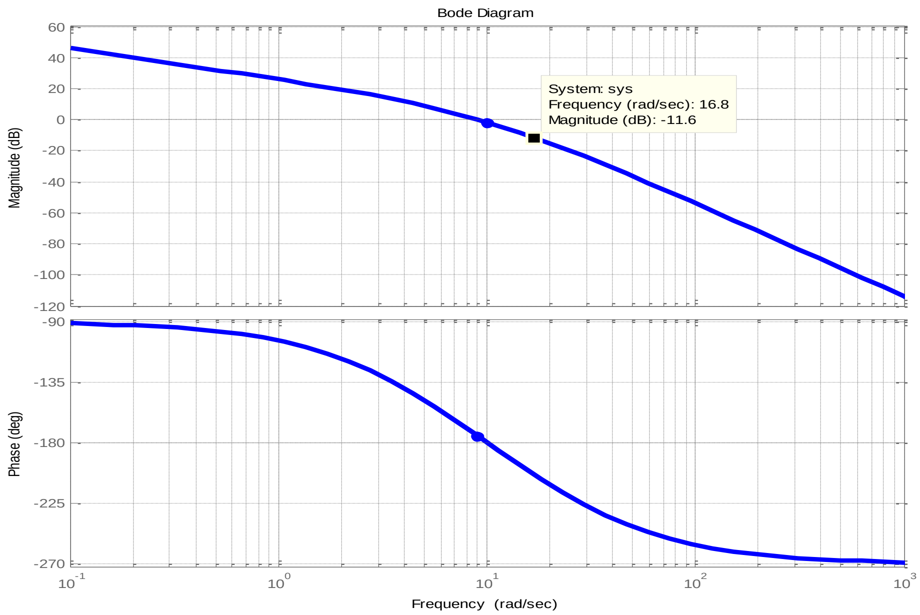
Step IV: Determine α

$$\varphi_{max} = 60 = sin^{-1}\frac{1-\alpha}{1+\alpha};$$
$$\alpha = 0.0718$$

Step V: From $-20\log\frac{1}{\sqrt{\alpha}} = -20\log\frac{1}{\sqrt{0.0718}} = -11.438$ dB at ω_{max}

Select ω_{max} to be new phase crossover frequency at this frequency must be – 11. 6 dB.

(approximately) at 16.8 rad/sec is ω_{max}



$$\omega_{max} = \frac{1}{T\sqrt{\alpha}} = 16.8 \quad rad/sec$$

Thus T= 0.222; $\alpha = 0.0718$

Now lead compensator is

$$G_c(s) = K_c\frac{s+1/T}{s+1/\alpha T} = K_c\left(\frac{s+4.5}{s+62.697}\right)$$

Step VI: Determine gain of lead compensator

$$K_c\alpha = K = 2000 \rightarrow K_c = 27855.153$$

Now lead compensator is $G_c(s) = 27855.153\left(\frac{s+4.5}{s+62.697}\right)$